Quiz 10, Linear

8:07
8:11
4
give 15 minutes.

Name: Key

(if any)

1. (4 points) Find the characteristic polynomial and eigenvalue(s) for the matrix $\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$.

$$\det\left(\begin{bmatrix} 2-\lambda & 1 \\ -1 & 4-\lambda \end{bmatrix}\right) = 0$$

$$(2-\lambda)(4-\lambda) - (-1) = 0$$

$$8 - 6\lambda + \lambda^2 + 1 = 0$$

$$\boxed{(\lambda-3)^2} = 0$$

char. polyn.

$\Rightarrow \boxed{\lambda = 3}$ is the only eigenvalue, has multiplicity 2.

2. (3 points) Let λ be an eigenvalue of an invertible matrix A. Show that λ^{-1} is an eigenvalue of A^{-1}. (Hint: Suppose a nonzero $\mathbf{x}$ satisfies $A\mathbf{x} = \lambda\mathbf{x}$.)

$Ax = \lambda x$ for some $x \neq 0$

$\Longleftrightarrow A^{-1}Ax = A^{-1}\lambda x$

$\Longleftrightarrow x = \lambda A^{-1}x$

$\Longleftrightarrow \lambda^{-1}x = A^{-1}x$ for some $x \neq 0$

$\Longrightarrow \lambda^{-1}$ is an eigenvalue of A^{-1}!

3. (3 points) A is a 4×4 matrix with three eigenvalues. One eigenspace is one-dimensional and one of the other eigenspaces is two-dimensional. Is it possible the A is *not* diagonalizable? Justify your answer.

1 1-dim'l, 1 2-dim'l, total dimension for A is 4, So the 3rd eigenvalue must have dimension 1. Thus the sum of the dimensions of the eigenspaces = 4, So A must be diagonalizable.